Fourth Semester B.Tech. Degree Examination, July 2015
(2008 Scheme)
Branch : Electronics and Communication
08.401 : ENGINEERING MATHEMATICS – III PROBABILITY AND RANDOM PROCESS (TA)

Time : 3 Hours Max. Marks : 100

***Instruction** : Answer **all** questions of Part – **A** and **one full** question from **each** Modules **I, II** and **III** of Part – **B**.*

PART – A

1. A random variable X has the following probability distribution

x :	–2	–1	0	1	2	3
p(x) :	0.1	k	0.2	2k	0.3	3k

(i) Find k (ii) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$.

2. Show that the mean and variance of a Poisson distribution are equal.

3. Define uniform distribution. Show that the probability density function of a uniform distribution in (a, b) is $f(x) = \frac{1}{b-a}$, $a < X < b$.

4. Let $X_1, X_2, \ldots X_{100}$ be independent identically distributed random variables with $\mu = 2$ and $\sigma^2 = \frac{1}{4}$. Use Central Limit Theorem to find $P(192 < X_1 + X_2 + \ldots + X_{100} < 210)$.

5. The joint density function of X and Y is

$$f(x, y) = \begin{cases} e^{-(x+y)}, & 0 \le x, \; y \le \infty \\ 0 \quad , & \text{otherwise} \end{cases}$$

Show that X and Y are independent.

6. Suppose that $\{X(t)\}$ is a random process with mean $\mu(t) = 3$ and autocorrelation $R(t_1, t_2) = 9 + 4e^{-0.2|t_1 - t_2|}$. Determine the mean and variance of the random variables $Z = X(5)$ and $W = X(8)$.

P.T.O.

7. Define Poisson process. Show that Poisson process is not a stationary process.

8. When is a random process said to be ergodic. Give an example for an ergodic process.

9. Define spectral density function. State any two properties of spectral density function.

10. Assume that a travelling salesman is at an integral point of the X-axis between the origin and at the point x = 3. He can take a unit step either to the left or to the right. If he goes right, the probability is 0.7 and if he goes to the left, the probability is 0.3. If he is at the origin then he takes a step to the right to reach at x = 1. Similarly, if he is at x = 3, then he takes a step to the left to reach x = 2. Find the transition probability matrix. **(10×4=40 Marks)**

PART – B

Module – I

11. a) Find the mean and variance of the uniform distribution in an interval (a, b).

If X is a uniformly distributed random variable with mean 1 and variance $\frac{4}{3}$, find P [X < 0].

b) The joint distribution of X_1 and X_2 is given by

$$f(x_1, x_2) = \frac{x_1 + x_2}{21}, x_1 = 1, 2, 3 \text{ and } x_2 = 1, 2.$$

i) Find the marginal distributions of X_1 and X_2.

ii) Find the conditional distribution of X_1 given $X_2 = 1$.

c) Fit a binomial distribution for the following data

x :	0	1	2	3	4	5	6	**Total**
f :	5	18	28	12	7	6	4	**80**

(7+7+6=20)

12. a) In an examination 44% of the candidates obtained marks below 55 and 6% got above 80. Assuming the distribution is normal, find the mean and standard deviation of the distribution.

b) If X is the number scored in a throw of a fair die; show that the Chebychev's inequality gives $P[|X - \mu| \geq 2.5] < 0.47$, where μ is the mean of X, while the actual probability is zero.

c) Two random variables X and Y have joint probability density function

$$f(x, y) = \begin{cases} \frac{xy}{96}, & 0 < x < 4, 1 < y < 5 \\ 0, & \text{elsewhere} \end{cases}$$

Find cov (X, Y). Are X and Y independent. **(7+7+6=20)**

Module – II

13. a) If $X(t) = 5 \cos (10t + \theta)$ and $Y(t) = 20 \sin (10t + \theta)$ where θ is a random variable uniformly distributed in $(0, 2\pi)$, prove that the process {X(t)} and {Y(t)} are jointly wide-sense stationary.

b) Messages arrive at a telegraph office in accordance with the laws of a Poisson process with a mean rate of 3 messages per hour.

i) What is the probability that no message will have arrived during the morning hours, i.e. between 8A.M. and 12 Noon.

ii) What is the distribution of the time at which the first afternoon message arrives ?

c) If {X(t)} is a SSS process, prove that E[X(t)] is a constant. **(7+7+6=20)**

14. a) Define evolutionary process. Show that the process {X(t)} such that

$$P[X(t) = n] = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & \text{if } n = 1, 2, ... \\ \frac{at}{1+at}, & \text{if } n = 0 \end{cases} \quad \text{is evolutionary.}$$

b) Establish the necessary and sufficient condition for the wide-sense stationarity of the process.

$X(t) = A \cos \omega t + B \sin \omega t$ where A and B are random variables.

c) A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. **(7+7+6=20)**

Module – III

15. a) Find the power spectral density of the random process, if its autocorrelation function is $R(\tau) = e^{-\alpha|\tau|} \cos\beta\tau$.

b) The transition probability matrix of a Markov chain $\{X_n\}$ having 3 states 0, 1, 2 is

$$P = \begin{bmatrix} 0.75 & 0.25 & 0 \\ 0.25 & 0.50 & 0.25 \\ 0 & 0.75 & 0.25 \end{bmatrix}$$

and the initial distribution is $P^{(0)} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

Find (i) $P[X_3 = 2, X_2 = 1, X_1 = 1, X_0 = 2]$ (ii) $P(X_2 = 2)$. **(10+10=20)**

16. a) The autocorrelation function of a WSS process is $R(\tau) = \rho e^{-\rho|\tau|}$, show that the spectral density function is $S(\omega) = \dfrac{2}{1+\left(\dfrac{\omega}{\rho}\right)^2}$.

b) If $\{X(t)\}$ is a WSS process with mean μ and autocovariance function

$$c(\tau) = \begin{cases} \sigma_x^2\left(1-\dfrac{|\tau|}{\tau_0}\right) & \text{for } 0 \le |\tau| \le \tau_0 \\ 0 & \text{for } |\tau| \ge \tau_0 \end{cases}$$

Find the variance of the time-average of $\{X(t)\}$ over $(0, T)$. Also examine if the process is mean-ergodic.

c) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is $\frac{1}{3}$ and the probability of a rainy day following a dry day is $\frac{1}{2}$. Given that May 1 is a dry day. Find the probability that

i) May 3 is also a dry day and

ii) May 5 is also a dry day. **(7+7+6=20)**

———————